Analytic Geometry

Question 3 [34 Marks]

a) Find the point of intersection and the angle between the pair of lines $x^2+xy-3x=0$.

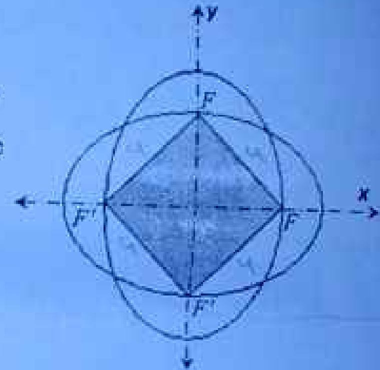
b) Find the equation of a circle whose center (2, 3) and cuts the circle $x^2+y^2=4$ orthogonally.

c) For the parabola $y^2-2y+8x=15$. Find the vertex, the focus and the ends points of latus rectum. Sketch this parabola.

d) Show that the equation of the rectangular (equilateral) hyperbola whose directrix $x+y=0$ and focus (0, 1) is $2xy+2y-1=0$. Find the conjugate hyperbola and the common asymptotes.

Question 4 [31 Marks]

a) The figure to the right shows two ellipses whose major axes are perpendicular to each other. Each ellipse passes through the other ellipse's foci (which form a square when connected, as shown). Given that the **square** encloses an area of **16**, and the center of two ellipses located at the origin, what is the equation of each ellipse?



b) Find the value (or values) of m such that the quadratic surface $x^2+m(y-1)^2+z^2=1$ represents: (i) Cylinder (ii) Sphere (iii) Ellipsoid (iv) Hyperboloid (v) Cone.

c) Find the equation of sphere in which the points $P_1(0,0,4)$ and $P_2(4,0,0)$ represent ends of diameter. Also, find: (i) The equation of another diameter which passes through $P_3(0,2,0)$.

(ii) The angle between the two diameters.

(iii) The equation of the plane through the three points $P_1 P_2 P_3$.

(iv) The intersection of sphere with the plane $y=2\sqrt{2}$. (Is this plane cuts, touches or not?)

d) The two circular paraboloids $x^2+y^2=z$ and $x^2+y^2=8-z$ intersected in a common circle. Find the area of this common circle.

Best of luck

تنبيه هام:

Question 1 [36 Marks]

(a) Evaluate the following integrals [24 Marks]

$$\int \frac{7x^2+3x}{(x-1)(x^2+4)}dx \qquad \int x\cot^{-1}x\,dx$$

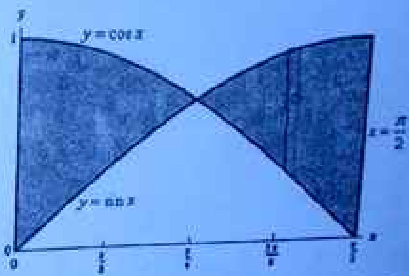
$$\int (x-1)^3\sqrt{x^2-2x+2}\,dx \qquad \int_{-\pi/2}^{\pi/2}\left[\tan^3 x+\frac{1}{2+\cos x}\right]dx$$

(b) Discuss the convergence and divergence of $\int_0^{\pi}\frac{dx}{1-\sin x}$ [6 Marks]

(c) Find the reduction formula for $I_{n+2}=\int(\ln x)^{n+2}dx$, state the allowable values of n, then find I_3. [6 Marks]

Question 2 [29 Marks]

(a) Find the average velocity of a particle if its acceleration is $\frac{dv}{dt}=\frac{1}{1+t}$, $v(0)=1$ for $0\le t\le 1$. Hint: the average value of the continuous function $v(t)$, $a\le t\le b$ is given by $\frac{1}{b-a}\int_a^b v(t)\,dt$. [7 Marks]



(b) For the area bounded by the curves

$y=\sin x$, $y=\cos x$, $x=\frac{\pi}{2}$, and y_axis

Find

i. The area of this region. [7 Marks]

ii. The volume of the solid generated by revolving this area about x_axis [7 Marks].

(c) Find the surface area of a cone storage tank, that is formed by revolving the line segment $x=4-4\sin^2 t$, $y=4\cos^2 t$ and $0\le t\le\frac{\pi}{2}$ about the y_axis [5 Marks]. If the outer surface of the storage tank including the circular lid (غطاء) is sprayed with a coat of paint. How many liters of paint will be used if each liter covers $10m^2$ [3 Marks].

اقلب الورقة من فضلك (جزء التحليلية)

Solution of 2012 integration test

$$\int \frac{7x^2+3x}{(x-1)(x^2+4)}dx = \int \frac{2}{x-1}+\frac{5x+8}{x^2+4}\,dx = 2\ln(x-1)+\frac{5}{2}\ln(x^2+4)+4\tan^{-1}\frac{x}{2}+k$$

$$\int x\cot^{-1}x\,dx = \frac{x^2}{2}\cot^{-1}x+\frac{1}{2}\int\frac{x^2}{x^2+1}dx = \frac{x^2}{2}\cot^{-1}x+\frac{1}{2}(x-\tan^{-1}x)+k$$

$$\int (x-1)^3\sqrt{x^2-2x+2}\,dx = \int (x-1)^3\sqrt{(x-1)^2+1}\,dx \overset{x-1=\sinh\theta}{\underset{or\ x-1=\tan\theta}{=}} \int \sinh^3\theta\cosh^2\theta\,d\theta = \frac{\cosh^5\theta}{5}-\frac{\cosh^3\theta}{3}+k$$

$$\int_{-\pi/2}^{\pi/2}\left[\tan^3x+\frac{1}{2+\cos x}\right]dx = \int_{-\pi/2}^{\pi/2}\tan^3x\,dx+\int_{-\pi/2}^{\pi/2}\frac{1}{2+\cos x}dx = 0 \overset{t=\tan x/2}{+} 4\int_0^1\frac{1}{t^2+3}dt = \frac{4\pi}{3\sqrt{3}}$$

$$\int_0^{\pi}\frac{dx}{1-\sin x} = \lim_{a\to\pi/2^-}\int_0^a\frac{dx}{1-\sin x}+\lim_{b\to\pi/2^+}\int_b^{\pi}\frac{dx}{1-\sin x} = \lim_{a\to\pi/2^-}\left[\sec x+\tan x\right]_0^a+\lim_{b\to\pi/2^+}\left[\sec x+\tan x\right]_b^{\pi}\Rightarrow div$$

$$I_{n+2}=x(\ln x)^{n+2}-(n+2)I_{n+1}\ ,\ n>-2,\ n=1\Rightarrow I_3=x(\ln x)^3-3I_2,\ldots complete\ ,$$

$$v=\int\frac{1}{1+t}dt=\ln(t+1)+c,\ v(0)=1\Rightarrow c=1,\ ave\ velocity=\frac{1}{1-0}\int_0^1\ln(t+1)+1\,dt=(t+1)\ln(t+1)|_0^1=2\ln 2$$

$$A=\int_0^{\pi/4}\cos x-\sin x\,dx+\int_{\pi/4}^{\pi/2}\sin x-\cos x\,dx=\frac{4}{\sqrt{2}}-2$$

$$V=V_1+V_2=2V_1=(2)*\pi\int_0^{\pi/4}\cos^2x-\sin^2x\,dx=(2)*\pi\int_0^{\pi/4}2\cos^2x-1dx=(2)*\pi\int_0^{\pi/4}\cos 2x\,dx=\pi$$

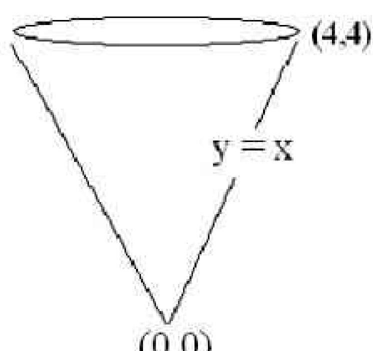
$$dL=\sqrt{(-8\sin t\cos t)^2+(-8\sin t\cos t)^2}=8\sqrt{2}\sin t\cos t\,dt$$

$$S=2\pi\int_0^{\pi/2}(4-4\sin^2t)8\sqrt{2}\sin t\cos t\,dt=16\sqrt{2}\pi\,m^2$$

$$4\sin^2t=4-x\quad(1)$$

$$4\cos^2t=y\qquad(2)$$

$$\overset{(1)+(2)}{\Rightarrow}\ 4(\sin^2t+\cos^2t)=4-x+y\Rightarrow y=x$$



$$Area of\ circle=\pi(4)^2=16\pi$$

$$Total\ area=16\sqrt{2}\pi+16\pi$$

$$Liters=\frac{16\sqrt{2}\pi+16\pi}{10}=1.6\pi(\sqrt{2}+1)$$